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Geodetic Number of Soft Graphs of Petersen Graph

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Abstract: Let G^* = (V, E) be a simple graph and A$\subseteq V(G^*)$ be any non-empty set of parameters. Let ρ be an arbitrary relation from A to V where (F, A) and (K, A) are soft sets over V and E respectively. H(a) = (F(a), K(a)) is an induced subgraph ofG^* for all a ∈A.Then,(G^*, F, K, A) = {H(a)/a ∈ A }≅ $\underset{a\in A}{\cup} H(a)$ is called the soft graph of G^* corresponding to the parameter set A and the relation ρ. It is said to be a T1- soft graph of G^*only if H(a) is connected ∀ a ∈ A.Otherwise it is called a T12-soft graph of G^*. Every T1-soft graph is also a T12-soft graph of G^* and not the converse. The geodetic set of (G^*, F, K, A)is introduced by K Palani et al.[7] and is defined as the union of geodetic sets of the induced sub graphs H(a) where a ∈ A. A geodetic set of a T1 or T12- soft graph of G^* of minimum cardinality is said to be a minimum geodetic set of (G^*, F, K, A). The geodetic number of (G^*, F, K, A)
is the cardinality of a minimum geodetic set of (G^*, F, K, A). The geodetic number of
(G^*, F, K, A) is denoted as g[(G^*, F, K, A)] whereas the geodetic number of any graph G is g(G).This paper analyses the soft graphs of Petersen graphand geodeticnumber of different soft graphs of Petersen graph.

Keywords: Geodetic Number, Soft Graph, Parameter, Petersen Graph.

1. Introduction

Molodtsov [5] introduced soft set theory in 1999 as a general mathematical tool for dealing with uncertainties. Maji, Biswas and Roy [4] made a theoretical study of the soft set theory in more detail.In2014, Rajesh K.Thumbakara and Bobin George [9] introduced the new notion soft graph using soft sets. In 2015, Akram M and Nawaz S [1] introduced the concept of soft graphs in broad spectrum.The soft graph has been studied in more detail in few papers. The geodetic sets of a connected graph were introduced by Frank Harary, Emmanuel Loukakis and Constantine Tsouros [2], as a tool for studying metric properties of connected graphs.Soft graphs of some standard and special graphs have been discussed in [6] and [8]. The geodetic set of (G^*, F, K, A) is introduced by K Palani et al. [7] and is defined as the union of geodetic sets of the induced sub graphs H (a) where a ∈ A

The **Petersen graph** is an undirected graph with 10 vertices and 15 edges. The Petersen graph is most commonly drawn as a pentagon with a pentagram inside, with five spokes. The Petersen graph is named after Julius Petersen, who in 1898 constructed it to be the smallest bridgeless cubic graph with no three-edge-colouring. In 1993, geodetic number of the Petersen graph is calculated in [2]. Here, the soft graph of Petersen graph is found and the geodetic number of various soft graphs of Petersen graph is evaluated. Also, geodetic number of soft graphs of edge- operated and vertex added Petersen graph is calculated .This work discusses the geodetic number of different soft graphs of the Petersen graph and so the Petersen graph is considered as G^*throughout.

2. Preliminaries

2.1 Definition: [9]

Let G^*= (V , E) be a simple graph and A be any non-empty set. Let $R \subseteq$ A × V be an arbitrary relation. A set valued function F:A→P(V)can be defined as F(x)={y∈V/ xRy}. The pair

(F, A) is a soft set over V. Then,(F,A)is said to be a **soft graph** of G^*, if the subgraph induced by F(x) in $G^*$is a connected subgraph of $G^*$for all $x \in A$. The set of all soft graph of G^* is denoted by SG(G^*).

2.2 Definition: [1]
Let G^* = (V, E) be a crisp graph and A be any non-empty set of parameters. Let R⊆ A×V be an arbitrary relation. A mapping F: A → P(V) can be defined as F(x) ={y ∈ V/x R y} and a mapping K: A → P(E) can be defined as K(x) = {uv∈ E/ {u, v} ⊆ F(x)}.
A 4-tuple G = (G^*, F, K, A) is called a **soft graph** of G^* if it satisfies the following properties:
(i) G^*= (V, E) is a simple graph
(ii) A is a nonempty set of parameters
(iii)(F, A) is a soft set over V
(iv)(K, A) is a soft set over E
(v)(F (a), K (a)) is a subgraph of G^* for all a ∈A
The subgraph (F (a), K (a)) is denoted by H (a). The set of all soft graphs of G^* is denoted by SG(G^*)

2.3 Definition:[7]
The geodetic set of the soft graph (G^*, F, K, A) is the union of geodetic sets of the induced subgraphs H(a), where a ∈A. A geodetic set of a minimum cardinality is said to be a minimum geodetic set of (G^*, F, K, A). The geodetic number of (G^*, F, K, A) is the cardinality of a minimum geodetic set of (G^*, F, K, A).

2.4 Remark: [2] The Petersen graph has geodetic number 4, determined by any set of four independent nodes

2.5 Notation: The geodetic number of (G^*, F, K, A) is denoted as g[(G^*, F, K, A)] whereas the geodetic of any graph G is g(G).

3. Soft Graphs of Petersen Graph

3.1 Observation:
1. Let A ⊆ $V(G^*)$ be any singleton parameter set. Define $\rho : A \to V$ by $x\ \rho\ y \Leftrightarrow$ d (x, y) ≤ 1. Then, (G^*, F, K, A) is isomorphic to $K_{1,3}$ and it is a T1- soft graph of G^*.
2. Let A ⊆ $V(G^*)$ be any t element parameter set. Define $\rho : A \to V$ by $x\ \rho\ y \Leftrightarrow$ d (x, y) ≤ 1. Then, (G^*, F, K, A) is isomorphic to $tK_{1,3}$ and is again a T1- soft graph of G^*
3. Let A ⊆ $V(G^*)$ be any singleton parameter set. Define $\rho : A \to V$ by $x\ \rho\ y \Leftrightarrow$ d (x, y) ≤ 1.The minimum geodetic sets corresponding to different possibilities of A are listed in the following table

Parameter set A	g-set
$\{v_1\}$	$\{v_2, v_5, w_1\}$
$\{v_2\}$	$\{v_1, v_3, w_2\}$
$\{v_3\}$	$\{v_2, v_4, w_3\}$
$\{v_4\}$	$\{v_3, v_5, w_4\}$
$\{v_5\}$	$\{v_4, v_1, w_5\}$
$\{w_1\}$	$\{v_1, w_3, w_4\}$
$\{w_2\}$	$\{v_2, w_4, w_5\}$
$\{w_3\}$	$\{v_3, w_5, w_1\}$
$\{w_4\}$	$\{v_4, w_1, w_2\}$
$\{w_5\}$	$\{v_5, w_2, w_3\}$

Table 3.1

From table 3.1, it is clear that g [(G^*, F, K, {v})] = 3∀ v ∈ V (G^*)

3.2 Theorem: Let A ⊆ $V(G^*)$ be any two-element parameter set. Define ρ :A ⟶V by
$x\ \rho\ y \Leftrightarrow d(x, y) \leq 1$. Then, 5 ≤g [($G^*$, F, K, A)]≤6

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Proof:
Let A= $\{x, y\}$

Case 1: x, y are any two adjacent v_i's or w_i's From table 3.1, H(x) and H(y) are isomorphic to $K_{1,3}$with disjoint geodetic sets. Therefore, g[(G^*, F, K, A)] = 6

Case 2: x, y are any two non-adjacent v_i's or w_i's Again, the table 3.1 shows that the geodetic sets of H(x) and H(y) have one common element and so, g[(G^*, F, K, A)] =5

Case 3: x, y are in different cycles Here, ifx, y are adjacent or non-adjacent, correspondingly from table 3.1,it is clear that g [(G^*, F, K, A)] is 5 or 6 From the above cases, 5 $\leq$ g [(G^*, F, K, A)] $\leq$ 6 if $|A|$ =2 and x $\rho\, y \Leftrightarrow d(x, y) \leq 1$.

3.3 Theorem: Let A$\subseteq V(G^*)$ be any three-element parameter set with all vertices from the outer pentagon. Then, g [(G^*, F, K, A)] = $\begin{cases} 8 & if\ \langle A \rangle = P_3 \\ 7 & otherwise \end{cases}$

Proof: The following table shows a parameter set A, its geodetic sets and geodetic number

A	Geodetic sets	$\langle A \rangle = P_3$	Geodetic number
$\{v_1, v_2, v_3\}$	$\{v_2, v_5, w_1\} \cup \{v_1, v_3, w_2\} \cup \{\boxed{v_2}, v_4, w_3\}$	Yes	8
$\{v_1, v_2, v_4\}$	$\{v_2, v_5, w_1\} \cup \{v_1, v_3, w_2\} \cup \{\boxed{v_3}, \boxed{v_5}, w_4\}$	No	7
$\{v_1, v_2, v_5\}$	$\{v_2, v_5, w_1\} \cup \{v_1, v_3, w_2\} \cup \{v_4, \boxed{v_1}, w_5\}$	Yes	8
$\{v_1, v_3, v_4\}$	$\{v_2, v_5, w_1\} \cup \{\boxed{v_2}, v_4, w_3\} \cup \{v_3, \boxed{v_5}, w_4\}$	No	7
$\{v_1, v_3, v_5\}$	$\{v_2, v_5, w_1\} \cup \{\boxed{v_2}, v_4, w_3\} \cup \{\boxed{v_4}, v_1, w_5\}$	No	7
$\{v_1, v_4, v_5\}$	$\{v_2, v_5, w_1\} \cup \{v_3, \boxed{v_5}, w_4\} \cup \{v_4, v_1, w_5\}$	Yes	8
$\{v_2, v_3, v_4\}$	$\{v_1, v_3, w_2\} \cup \{v_2, v_4, w_3\} \cup \{\boxed{v_3}, v_5, w_4\}$	Yes	8
$\{v_2, v_3, v_5\}$	$\{v_1, v_3, w_2\} \cup \{v_2, v_4, w_3\} \cup \{\boxed{v_4}, \boxed{v_1}, w_5\}$	No	7
$\{v_2, v_4, v_5\}$	$\{v_1, v_3, w_2\} \cup \{\boxed{v_3}, v_5, w_4\} \cup \{v_4, \boxed{v_1}, w_5\}$	No	7
$\{v_3, v_4, v_5\}$	$\{v_2, v_4, w_3\} \cup \{v_3, v_5, w_4\} \cup \{\boxed{v_4}, v_1, w_5\}$	Yes	8

Table 3.2 The proof follows from table 3.2

3.4 Remark: The following results are verified from similar tables for the concerned cases.
(i) If A contains all the three vertices from inner pentagram, then 7≤g [(G^*, F, K, A)] $\leq$ 8
(ii)If A contains mixed elements of pentagon and pentagram , then 6 ≤g [(G^*, F, K, A)]$\leq$ 8

3.5 Observations: Define ρ :A ⟶V by x $\rho\, y \Leftrightarrow d(x, y) \leq 1$
(i)If $|A| = 4$, then g[(G^*, F, K, A)]takes any integer from 5 to10 except 7
(ii)If $|A|$=5, then , 8 $\leq g[(G^*, \text{F}, \text{K}, \text{A})] \leq 10$
(iii)If $6 \leq |A| \leq 7$, then 9 $\leq$ g $[(G^*, \text{F}, \text{K}, \text{A})] \leq 10$
(iv)If $8 \leq |A| \leq 10$, then g[(G^*, F, K, A)]= 10

3.6 Theorem: Let A $\subseteq$V(G^*) be any t-element parameter set. Define ρ :A ⟶V by
x $\rho\, y \Leftrightarrow d(x, y) \leq 2$.Then, g[($G^*$, F, K, A)] = 4
Proof:

Here, H(a) for each element a of the parameter set is G^* itself and is of type T1SG(G^*).
The geodetic set of G^* is determined by any set of four independent nodes.Then,
g(G^*)= 4 which results in g[(G^*, F, K, A)] = 4

3.7 Observation:
Let A ⊆ $V(G^*)$ be any singleton parameter set.

1. Define $\rho : A \to V$ by x ρ y ⟺ d (x, y) = 1.Then, (G^*, F, K, A) is isomorphic to $\overline{K_3}$ and hence, disconnected .Further, g[(G^*, F, K, {v})] =3 ∀ v ∈ V(G^*)
2. Define $\rho : A \to V$ by x ρ y ⟺ d (x, y) = 2.Then, (G^*, F, K, A) is isomorphic to C_6 and hence, g [(G^*, F, K, {v})] =2 ∀ v ∈ V(G^*)

4. Geodetic number of soft graphs of an edge removed Petersen graph

4.1 Theorem: Consider a Petersen graphG^*. Let e ∈ $E(G^*)$ *and* A ⊆V(G^*) be a singleton parameter set. Define $\rho : A \to V$ by $x\rho\, y$ ⟺ d (x, y) ≤1. Then ,
g ((G^*, F, K, {x})]−1 ≤ g [($G^* - e$, F, K, {x})] ≤ g [(G^*, F, K, {x})] ∀ x ∈ V(G^*)
Proof:
Let e= uv and A ={x}
Here, ($G^* - e$, F, K, {x}) is isomorphic to P_3 at u , v and $K_{1,3}$ at all other vertices. The following table depicts the geodetic sets of ($G^* - e$, F, K, {x}).

A	g-sets		
	($G^* - v_1v_2$, F, K,A)	($G^* - w_1w_3$,F, K, A)	($G^* - w_1v_1$,F, K, A)
$\{v_1\}$	$\{v_5,w_1\}$	$\{v_2,v_5,w_1\}$	$\{v_2,v_5\}$
$\{v_2\}$	$\{v_3,w_2\}$	$\{v_1,v_3,w_2\}$	$\{v_1,v_3,w_2\}$
$\{v_3\}$	$\{v_2,v_4,w_3\}$	$\{v_2,v_4,w_3\}$	$\{v_2,v_4,w_3\}$
$\{v_4\}$	$\{v_3,v_5,w_4\}$	$\{v_3,v_5,w_4\}$	$\{v_3,v_5,w_4\}$
$\{v_5\}$	$\{v_4,v_1,w_5\}$	$\{v_4,v_1,w_5\}$	$\{v_4,v_1,w_5\}$
$\{w_1\}$	$\{v_1,w_3,w_4\}$	$\{v_1,w_4\}$	$\{w_3,w_4\}$
$\{w_2\}$	$\{v_2,w_4,w_5\}$	$\{v_2,w_4,w_5\}$	$\{v_2,w_4,w_5\}$
$\{w_3\}$	$\{v_3,w_5,w_1\}$	$\{v_3,w_5\}$	$\{v_3,w_5,w_1\}$
$\{w_4\}$	$\{v_4,w_1,w_2\}$	$\{v_4,w_1,w_2\}$	$\{v_4,w_1,w_2\}$
$\{w_5\}$	$\{v_5,w_2,w_3\}$	$\{v_5,w_2,w_3\}$	$\{v_5,w_2,w_3\}$

Table 4.1

From table 4.1, g [($G^* - e$, F, K, {x})] $=\begin{cases} 2 & \textit{if x is incident with e} \\ 3 & \textit{otherwise} \end{cases}$
It is checked that similar is the case with removal of any edge from G^*.
By observation 3.1, g [(G^*, F, K, {x})] =3
Hence g [(G^*, F, K, {x})]−1≤ g [($G^* - e$, F, K, {x})] ≤ g [(G^*, F, K, {x})]

4.2 Observation: Consider G^*. Let e ∈ $E(G^*)$. Define $\rho : A \to V$ by $x\rho\, y$ ⟺ d (x, y) ≤1

1. Let A= {x, y}⊆ $V(G^*)$ be the parameter set.Then, 4≤g [($G^* - e$, F, K, A)] ≤6n Further, g [($G^* - e$, F, K, A)] ≤g [(G^*, F, K, A)]
2. Let A⊆ $V(G^*)$be any 3-element parameter set.Then,5≤g [($G^* - e$, F, K, A)]≤ 8. Also, g [($G^* - e$, F, K, A)] ≤g [(G^*, F, K, A)].
3. If A is any t-element parameter set, g [($G^* - e$, F, K, A)] ≤g [(G^*, F, K, A)]

5. Geodetic number of soft graphs of a vertex added Petersen graph

5.1 Theorem: Let G^* be the Petersen graph.Let $A = \{x\} \subseteq V(G^*)$ be the parameter set. Define $\rho : A \to V$ by x ρy ⟺ d (x, y) ≤1 .Then,
g [(G^*, F, K, A)]≤ g[(G^* + v, F, K, A)] ≤g [(G^*, F, K, A)]+1.
Proof:
Attach a new vertex v to v_1 ∈ V(G^*) . Let $A = \{x\}$be the parameter set and given

$x\,\rho y$ ⟺ d (x, y) ≤1 . Also from observation 3.1, g [(G^*, F, K, A)] = 3
Suppose the new vertex v is attached to a vertex v_1 of G^*. The corresponding g- sets for singleton parameter set after adding a vertex are given in the following table.

A	g - sets
$\{v_1\}$	$\{v_2, v_5, w_1, v\}$
$\{v_2\}$	$\{v_1, v_3, w_2\}$
$\{v_3\}$	$\{v_2, v_4, w_3\}$
$\{v_4\}$	$\{v_3, v_5, w_4\}$
$\{v_5\}$	$\{v_4, v_1, w_5\}$
$\{w_1\}$	$\{v_1, w_3, w_4\}$
$\{w_2\}$	$\{v_2, w_4, w_5\}$
$\{w_3\}$	$\{v_3, w_5, w_1\}$
$\{w_4\}$	$\{v_4, w_1, w_2\}$
$\{w_5\}$	$\{v_5, w_2, w_3\}$

Table 5.1

From table 5.1, g[($G^* + v$, F, K, A)] = 3 or 4 whereasg [(G^*, F, K, A)] = 3
If v is attached to any vertex 'x' other than v_1 , the change happens only in the geodetic sets of v_1 , and x in the above table. That is, for v_1 , a 3-element set and for x, a 4-element set. Hence,
g[($G^* +$ v, F, K, A)] = 3 or 4
Therefore,g[(G^*, F, K, A)]≤ g[($G^* +$ v, F, K, A)] ≤g [(G^*, F, K, A)]+1

5.2 Theorem: Let A = $\{x, y\}$⊆ $V(G^*)$ be the parameter set. Define $\rho : A \to V$ by $x\rho\, y$ ⟺ d (x, y) ≤1. Then, 5 ≤ g [($G^* + v$, F, K, A)] ≤ 7.
Proof:
Let A = {x, y} be the parameter set and given xρy ⟺ d (x, y) ≤1
As in the above theorem, if v is attached to any vertex 'x' other than v_1 , the change happens only in the geodetic sets of v_1 , and x and hence g[($G^* +$ v, F, K, A)] is 5 or 6 or 7.
Then, 5 ≤ g [($G^* + v$, F, K, A)] ≤ 7

5.3 Observation: If A beany t-element parameter set, then
g [($G^* + v$, F, K, A)]≥g [(G^*, F, K, A)]

6. Conclusion

This paper evaluates the geodetic number of soft graphs arising from Petersen graph and the changes due to removal of an edge or addition of a vertex.

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