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Thermal Effects in Nanofluid Flow with Free Convection Boundary Layer

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Abstract: This paper outlines the derivation of conservation equations relevant to nanofluid dynamics in the absence of a solid matrix and extends the analysis to include porous media saturated by the nanofluid in the presence of a heat source. Utilizing a Darcy model for the momentum equation, our study investigates the potential impact of introducing a Brinkman term on the qualitative aspects of the equations. Through this investigation, we aim to provide insights into the behavior of nanofluids within porous media under the influence of thermal effects.

Keywords: Nanofluid, conservation equations, heat source, qualitative analysis.

1. Introduction

Nanofluids, colloidal suspensions comprising nanoparticles dispersed in a base fluid, have garnered significant attention in recent years due to their unique thermal and fluidic properties. These properties, arising from the interaction between nanoparticles and the base fluid, offer promising avenues for enhancing heat transfer efficiency and fluid flow characteristics in various engineering applications. From thermal management systems to advanced cooling technologies, the utilization of nanofluids presents an opportunity to address challenges in heat transfer and thermal regulation across diverse industries [1]-[3]. Research efforts in the field of nanofluids have been directed towards understanding their behavior under different conditions and environments. Central to this research endeavor is the derivation and analysis of conservation equations governing the transport phenomena associated with nanofluid flow and heat transfer. These equations serve as the cornerstone for theoretical modeling and numerical simulations aimed at predicting and optimizing nanofluid behavior in real-world scenarios [3]-[6].

Nanotechnology opens a novel realm of investigation focused on processing and manufacturing materials with crystallite sizes averaging below 100nm, termed nanomaterials [4]. Within the spectrum of nanomaterials, one encounters a diverse array of substances, including nanocrystalline materials, nanocomposites, carbon nanotubes, and quantum dots. On the other hand, the term "nanofluid" denotes a liquid containing a dispersion of submicron solid particles, commonly referred to as nanoparticles. The concept of nanofluids has garnered attention due to its distinctive characteristic of thermal conductivity enhancement [5]. This remarkable phenomenon has spurred discussions regarding potential applications in advanced nuclear systems. Furthermore, recent investigations have explored the use of nanofluid flow in nano-drug delivery, showcasing the versatility and promise of nanofluid technology in various domains [6]-[9].

An investigation into convective transport phenomena in nanofluids, highlighting the persistent challenge of elucidating the abnormal increase in thermal conductivity and viscosity observed in these fluids [10]. Despite extensive research, a satisfactory explanation for this phenomenon remains elusive. While some scholars have posited that the dispersion of nanoparticles may contribute to convective heat transfer enhancement, Furthermore, the study dismisses turbulence and particle rotation as significant factors influencing heat transfer enhancement, as empirical calculations suggest their effects are negligible. In light of these findings, Buongiorno proposes a novel model grounded in the mechanics of nanoparticle/base-fluid relative velocity as a potential explanation for the observed enhancements in convective heat transfer.

The onset of convection in a horizontal layer uniformly heated from below, specifically focusing on its manifestation in nanofluids, building upon the transport equations proposed in the literature [11]. In this project, we extend this inquiry to explore the analogous problem in the context of flow through a porous medium, known as the Horton-Rogers-Lapwood problem. We posit the presence of suspended nanoparticles in the nanofluid, facilitated by either surfactant or surface charge technology, to prevent particle agglomeration and deposition on the porous matrix [10]-[13].
For thoroughness, it is worth noting that the study [14]-[15] provided a significantly different approach to addressing the Bénard problem in the context of nanofluids. In their work, Kim et al. [11] opted to modify three key quantities within the definition of the Rayleigh number: the thermal expansion coefficient, the thermal diffusivity, and the kinematic viscosity. This departure from conventional treatments underscores the diverse methodologies employed in exploring convective phenomena in nanofluids.

The modeling of a microchannel heat sink utilizing a nanofluid within a porous medium framework. Additionally, previous studies have examined convection within porous media, incorporating the phenomenon of thermophoresis particle deposition, as demonstrated by research such as that conducted by [10]- [15]. However, it is crucial to note that a distinguishing characteristic of nanofluids is their ability to mitigate particle deposition through specialized treatments, effectively rendering it negligible. This feature underscores the unique properties and potential applications of nanofluids in heat transfer and fluid dynamics contexts.

Similarly, it seems that research focusing on Brownian motion within porous media primarily pertains to deposition phenomena, which may not directly align with the scope of the current investigation. In our study, we have instead built upon the earlier work, extending their findings to further explore the dynamics and behavior of nanofluids in porous media under various conditions. This extension allows us to delve deeper into the intricate interplay between nanofluid flow, heat transfer, and porous media characteristics, offering valuable insights into this complex and important area of research.

This paper is structured as follows. An overview of nanoparicles, nonofulid applications in modern mathematical studies are discussed in section I. Section II presents the Pertubation soluation for nonofluid flow. Results and discussions are presented in section III. Finally, the conclusion of the work is given in section IV.

Solution of Perturbation for Nanofluid flow

This section presents a superimpose perturbations solution. The superimpose perturbations for a basic solution is given by equation .1.

$$\left.\begin{aligned} V &= V^1 \\ p &= p_b + p' \\ T &= T_b + T' \\ \emptyset &= \emptyset_b + \emptyset' \end{aligned}\right\} \quad (1)$$

A highly simplified equation from the (1) is given by (2)-(5).

$$0 = -\nabla p' - V' + RaT'\hat{e}_z - Rn\phi'\hat{e}_z \quad (2)$$

$$\frac{\partial T'}{\partial t} - w' = \nabla^2 T' + \frac{N_B}{Le}\left(\frac{\partial T'}{\partial z} - \frac{\partial \phi'}{\partial z}\right) - \frac{2N_A N_B}{Le}\frac{\partial T'}{\partial z} + \alpha(T_b + T') \quad (3)$$

$$\frac{1}{\sigma}\frac{\partial \phi'}{\partial t} + \frac{1}{\varepsilon}w' = \frac{1}{Le}\nabla^2\phi' + \frac{N_A}{Le}\nabla^2 T' \quad (4)$$

$w' = 0, \quad T' = 0, \quad \phi' = 0,$ at $z = 0,$ and at $z = 1.$ (5)

One should note that some parameters are not involved in these and subsequent equations and those are just a measure of the basic static pressure gradient. For the case of a regular fluid (not a nanofluid) the parameters $Rn,$ N_A and N_B are zero, the second term in Eq. (4) is absent because of $\partial\phi / \partial z = 0$ and then Eq. (5) is satisfied trivially. The remaining equations are reduced to the familiar equations for the Horton–Roger–Lapwood problem.

The six unknowns, such as $u', v', w', p', T', \phi'$ from the equations can be reduced to three by operating on Eq. (2) with $\hat{e}_z \cdot curlcurl$ and using the identity property.

$$curlcurl = graddiv - \nabla^2$$

$$\nabla^2 w' = Ra\nabla_H^2 T' + Rn\nabla_H^2 \phi' \quad (6)$$

In (6), the ∇_H^2 is the two-dimensional Laplacian operator for the given parameters on the horizontal plane. Further, method of normal modes is useful for the solution of the differential equations and the boundary conditions, which constitute a linear boundary-value problem. Now, we can write the (7).

$$(w', T'\phi') = [W(z), \Theta(z), \Phi(z)]\exp(st + ilx + imy) \quad (7)$$

and substitute into the differential equations to obtain (8)

$$(D^2 - \gamma^2)W + Ra\gamma^2\Theta - Rn\gamma^2\Phi = 0, \quad (8)$$

$$W + \left(D^2 + \frac{N_B}{Le}D - \frac{2N_A N_B}{Le}D - \gamma^2 - s + \alpha\right)\Theta - \frac{N_B}{Le}D\phi + \alpha(T_b - T') = 0, \quad (9)$$

$$\frac{1}{\varepsilon} - \frac{N_A}{Le}(D^2 - \gamma^2)\Theta - \left(\frac{1}{Le}(D^2 - \gamma^2) - \frac{s}{\sigma}\right)\Phi = 0, \quad (10)$$

$$W = 0, \Theta = 0, \Phi = 0, z = 0, z = 1, \quad (11)$$

In (11),

$$D = \frac{d}{dz}, \gamma = (l^2 + m^2)^{1/2}. \quad (12)$$

Thus γ is a dimensionless horizontal wave number.

For neutral stability the real part of s is zero. Hence, we can write $s = i\omega,$, where ω is real and is a dimensionless of frequency.

Galerkin-type weighted residuals method can be applied to obtain an approximate solution to the system of Eqs. (8)-(12). We choose as trial functions (satisfying the boundary conditions)

$$W_p = \Theta_p = \Phi_p = \sin p\pi z;\ p = 1,2,3.........$$

$$W = \sum_{p=1}^{N} A_p W_p, \Theta = \sum_{p=1}^{N} B_p \Theta_p, \Theta = \sum_{p=1}^{N} C_p \Theta_p \tag{13}$$

Now one can substitute the equations and simplify of (9) -(11) and make the expressions on the LHS (left hand side) of the expressions of the residuals orthogonal to the trial functions. The resultant is a system of three-N linear algebraic equations in the $3N$ unknowns $A_p, B_p, C_p;\ p = 1,2.....N$. The vanishing of the determinant of coefficients produces the eigenvalue equation for the system and one can note that the R_a is an eigenvalue. Thus Ra is found in terms of the other parameters.

2. Results and Discussions

A sketch of $Rn\left(N_A + \frac{Le}{\varepsilon}\right)$ versus Ra is given in Fig. 1. The sketch is made on the assumption that $\left(\varepsilon N_A + Le\right)/\sigma$ is greater than unity. If that inequality is reversed than the labels on the axes need to be swapped around.

The literature of references [9]-[12] suggests that the analysis predicts a significant reduction in the critical Rayleigh number for the bottom-heavy case, whereas our analysis indicates an increase in the critical Rayleigh number for non-oscillatory instability in this scenario. Notably, Tzou et al. [9] does not provide a physical explanation for this substantial reduction. It is worth mentioning that Tzou employs the symbol Le to represent a Lewis number divided by the nanoparticle fraction decrement, rather than a regular Lewis number. Consequently, the parameter Le tends to infinity as the nanoparticle fraction decrement tends to zero, i.e., in the limit as the nanofluid is replaced by a regular fluid. Based on this observation, we hypothesize that the solution obtained.

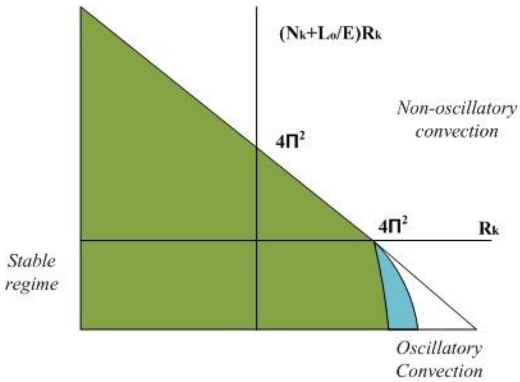

Figure 1: Stability and instability domains of the proposed study.

3. Conclusion

In this study, we have implemented a Darcy model to describe the momentum equation. We anticipate that introducing a Brinkman term into this equation will not yield significant qualitative alterations. Instead, we expect that the value denoted by 40 will be replaced by a larger value, Ra_0,contingent upon the hydrodynamic boundary conditions, and will escalate with the augmentation of the Darcy number. Consequently, the increase in this value implies that the fluctuation in the value of Ra_0, for a constant value of Rn, diminishes proportionately to the value of Ra_0. For instance, transitioning from free-free boundary conditions to more restrictive rigid-rigid boundary conditions, which augments the value, results in a reduction in the sensitivity to a given alteration.

4. References

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